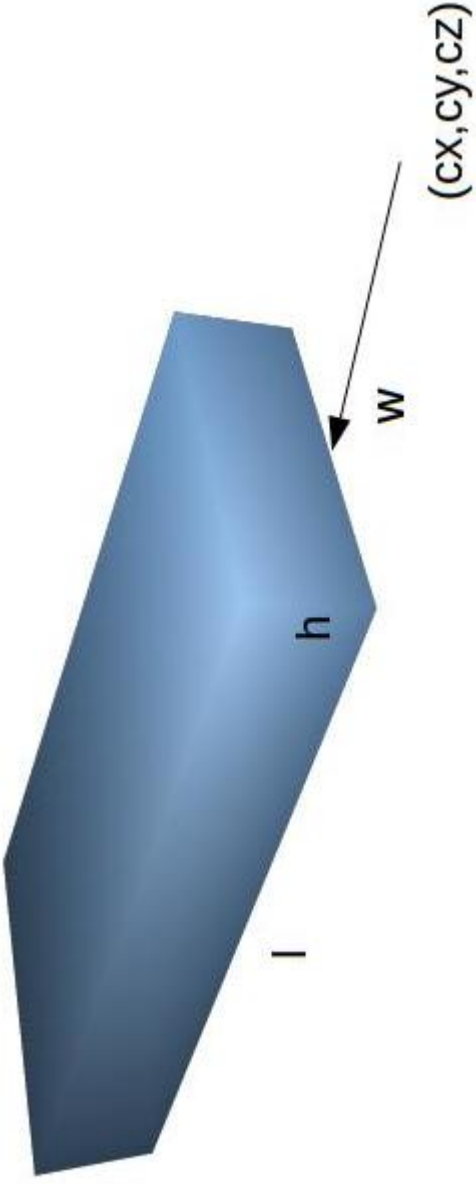


Infinitely Long Road, One Car

We prove that for the version of the game where the road is infinitely long and there is one car, the chicken has a strategy to cross the road in a finite amount of time.

Remember, the chicken is treated as an infinitesimal point, the car is simplified to have a box shape, and the car's position is taken to be the midpoint of the segment of the front of the car that lies at the base of the car. See the image below for an illustration.



We introduce some notation here. Denote H to be the width of the road, w to be the width of the car, h to be the height of the car l to be the length of a car, and T to be the maximum reaction time for the chicken. Let k_x, k_y, k_z be the x, y , and z positions of the chicken, respectively. Let v_x, v_y, v_z be the x, y , and z components of the velocity of the chicken, respectively. Let C be a newly defined sort of cars. Let c_x, c_y, c_z be the x, y , and z positions of the car. Let u_x be the x component of the velocity of the car (remember, the car only get to move in the x -direction. The ground coincides with the $x - y$ plane.

q is some auxiliary variable.

We introduce some abbreviations:

$$\begin{aligned}
A &\equiv k_x = 0 \wedge k_y = 0 \wedge k_z = 0 \wedge v_x = 0 \wedge v_y = 0 \wedge v_z = 0 \wedge 0 < c_y - \frac{w}{2} \wedge c_y + \frac{w}{2} < H \wedge c_z = 0 \wedge u_x > 0 \\
B &\equiv H > 0 \wedge w > 0 \wedge h \geq 0 \wedge l \geq 0 \wedge T > 0 \\
\alpha &\equiv k_z := h + 1; v_x := 0; v_y := 0; v_z := 0 \\
\beta &\equiv k_z := 0; v_x := *; v_y := *; v_z := 0; ?(v_y > 0); \gamma_1; \gamma_2; \text{if } v_x = u_x \text{ then } ?(k_x < c_x - l \vee c_x < k_x \vee t_1 > T) \text{ else } \gamma_3; \gamma_4; \delta_1; \delta_2 \text{ fi} \\
\gamma_1 &\equiv t_1 := \frac{c_y - \frac{w}{2} - k_y}{v_y} \\
\gamma_2 &\equiv t_2 := \frac{c_y + \frac{w}{2} - k_y}{v_y} \\
\gamma_3 &\equiv s_1 := \frac{c_x - k_x}{v_x - u_x} \\
\gamma_4 &\equiv s_2 := \frac{c_x - l - k_x}{v_x - u_x} \\
\delta_1 &\equiv \text{if } s_1 > s_2 \text{ then } q := s_1; s_1 := s_2; s_2 := q \text{ fi} \\
\delta_2 &\equiv ?(t_2 < s_1 \vee s_2 < t_1 \vee t_1 > T \vee s_1 > T) \\
\epsilon &\equiv \{c'_x = u_x, k'_x = v_x, k'_y = v_y, k'_z = v_z, t' = 1, t \leq T\} \\
\chi &\equiv t := 0; (\alpha \cup \beta); \epsilon
\end{aligned}$$

Then we wish to prove the following:

$$A \wedge B \rightarrow \langle \chi^* \rangle k_y \geq H$$

This is the same as the program found in the key file `chick_1car_ext.key`. We will prove this in multiple steps:

Let $p(\mathbf{k}, \mathbf{c})$ be the predicate $\langle \chi^* \rangle k_y \geq H$, where the vectors in the arguments have their obvious meanings.

$$\begin{array}{c}
\dots \quad Y_1 \\
\begin{array}{c}
(\cdot), (\cdot :=), (\cup) \quad \frac{\dots \quad \forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee \{t := 0\} \langle \alpha \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c}) \vee \{t := 0\} \langle \beta \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})) \rightarrow p(\mathbf{k}, \mathbf{c})) \rightarrow (A \wedge B \rightarrow p(\mathbf{k}, \mathbf{c}))}{\forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee \langle \chi \rangle p(\mathbf{k}, \mathbf{c})) \rightarrow p(\mathbf{k}, \mathbf{c})) \rightarrow (A \wedge B \rightarrow p(\mathbf{k}, \mathbf{c}))} \\
US \quad \frac{\forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee \langle \chi \rangle \langle \chi^* \rangle k_y \geq H) \rightarrow \langle \chi^* \rangle k_y \geq H) \rightarrow (A \wedge B \rightarrow \langle \chi^* \rangle k_y \geq H)}{A \wedge B \rightarrow \langle \chi^* \rangle k_y \geq H} \\
(*), \forall, MP
\end{array}
\end{array}$$

Let

$$\begin{aligned}
X_1 &\equiv \{t := 0\} \langle \alpha \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c}) \\
X_2 &\equiv \{t := 0\} \langle \beta \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})
\end{aligned}$$

Reduction of X_1 :

Let

$$\zeta_1 \equiv c_x := c_x + u_x \cdot \tilde{r}; k_x := k_x + v_x \cdot \tilde{r}; k_y := k_y + v_y \cdot \tilde{r}; k_z := k_z + v_z \cdot \tilde{r}; t := t + \tilde{r}$$

$$\zeta_2 \equiv c_x := c_x + u_x \cdot r; k_x := k_x + v_x \cdot r; k_y := k_y + v_y \cdot r; k_z := k_z + v_z \cdot r; t := t + r$$

$$\begin{array}{c} \mathbb{R} \frac{\exists 0 \leq r \leq T. p((k_x, k_y, h+1), (c_x + u_x \cdot r, c_y, c_z))}{\exists r \geq 0 ((\forall 0 \leq \tilde{r} \leq r \ \tilde{r} \leq T) \wedge p((k_x, k_y, h+1), (c_x + u_x \cdot \tilde{r}, c_y, c_z)))} \\ \text{subst} \frac{\{t := 0; k_z := h+1; v_x := 0; v_y := 0; v_z := 0\} \exists r \geq 0 ((\forall 0 \leq \tilde{r} \leq r \ \langle \zeta_1 \rangle t \leq T) \wedge \langle \zeta_2 \rangle p(\mathbf{k}, \mathbf{c}))}{\langle' \rangle} \\ \hline \langle; \rangle, \langle := \rangle \frac{\{t := 0; k_z := h+1; v_x := 0; v_y := 0; v_z := 0\} \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})}{\{t := 0\} \langle \alpha \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})} \end{array}$$

Reduction of X_2 :

Let

$$\zeta_3 \equiv t := 0; k_z := 0; v_x := \tilde{v}_x; v_y := \tilde{v}_y; v_z := 0$$

$$\zeta_4 \equiv t := 0; k_z := 0; v_x := \tilde{v}_x; v_y := \tilde{v}_y; v_z := 0; t_1 := \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y}; t_2 := \frac{c_y + \frac{w}{2} - k_y}{\tilde{v}_y}$$

$$\zeta_5 \equiv ?(k_x < c_x - l \vee c_x < k_x \vee t_1 > T)$$

ω

$$\begin{array}{c} \text{if, sub} \frac{\text{if, sub} \frac{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge ((\tilde{v}_x = u_x \wedge \{\zeta_4\} \langle \zeta_5 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})) \vee (\tilde{v}_x \neq u_x \wedge \{\zeta_4\} \langle \gamma_3; \gamma_4; \delta_1; \delta_2 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})))}{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \{\zeta_4\} \langle \text{if } v_x = u_x \text{ then } \zeta_5 \text{ else } \gamma_3; \gamma_4; \delta_1; \delta_2 \text{ fi} \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})}}{\exists \tilde{v}_x, \tilde{v}_y. \{\zeta_3\} \langle ?(v_y > 0) \rangle \langle \gamma_1 \rangle \langle \gamma_2 \rangle \langle \text{if } v_x = u_x \text{ then } \zeta_5 \text{ else } \gamma_3; \gamma_4; \delta_1; \delta_2 \text{ fi} \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})}} \\ \langle ? \rangle, \langle := \rangle, \text{sub} \frac{\{t := 0\} \langle \beta \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})}{\langle; \rangle, \langle := \rangle, \langle; * \rangle} \end{array}$$

where \tilde{v}_x and \tilde{v}_y are fresh, and where

$$Z_1 \equiv \exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x = u_x \wedge \{\zeta_4\} \langle \zeta_5 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})$$

$$Z_2 \equiv \exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \{\zeta_4\} \langle \gamma_3; \gamma_4; \delta_1; \delta_2 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})$$

Reduction of Z_1 :

Let

$$F_1 \equiv \tilde{v}_y > 0 \wedge \tilde{v}_x = u_x \wedge \left(k_x < c_x - l \vee c_x < k_x \vee \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} > T \right)$$

$$\begin{array}{c}
ax \frac{\exists \tilde{v}_x, \tilde{v}_y, 0 \leq r \leq T. (F_1 \wedge p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z)))}{\exists \tilde{v}_x, \tilde{v}_y. (F_1 \wedge \exists 0 \leq r \leq T. p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z)))} \\
\langle' \rangle, sub, \mathbb{R} \frac{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x = u_x \wedge \left(k_x < c_x - l \vee c_x < k_x \vee \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} > T \right) \wedge \{\zeta_4\} \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})}{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x = u_x \wedge \{\zeta_4\} \langle \zeta_5 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})} \\
\langle ? \rangle, \langle := \rangle, sub
\end{array}$$

Reduction of Z_2 :

Let

$$\zeta_6 \equiv t := 0; k_z := 0; v_x := \tilde{v}_x; v_y := \tilde{v}_y; v_z := 0; t_1 := \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y}; t_2 := \frac{c_y + \frac{w}{2} - k_y}{\tilde{v}_y}; s_1 := \frac{c_x - k_x}{\tilde{v}_x - u_x}; s_2 := \frac{c_x - l - k_x}{\tilde{v}_x - u_x}$$

$$\begin{array}{c}
Z_3 \vee Z_4 \\
if, sub, \vee \frac{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \{\zeta_6\} \langle \delta_1 \rangle \langle \delta_2 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})}{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \{\zeta_4\} \langle \gamma_3; \gamma_4; \delta_1; \delta_2 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})} \\
\langle : \rangle, \langle := \rangle, sub
\end{array}$$

where

$$\begin{aligned}
Z_3 &\equiv \exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \left(\frac{c_x - k_x}{\tilde{v}_x - u_x} > \frac{c_x - l - k_x}{\tilde{v}_x - u_x} \right) \wedge \{\zeta_6\} \langle q := s_1; s_1 := s_2; s_2 := q \rangle \langle \delta_2 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c}) \\
Z_4 &\equiv \exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \left(\frac{c_x - k_x}{\tilde{v}_x - u_x} \leq \frac{c_x - l - k_x}{\tilde{v}_x - u_x} \right) \wedge \{\zeta_6\} \langle \delta_2 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})
\end{aligned}$$

Reduction of Z_3 :

Let

$$\begin{aligned}
\zeta_7 &\equiv t := 0; k_z := 0; v_x := \tilde{v}_x; v_y := \tilde{v}_y; v_z := 0; t_1 := \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y}; t_2 := \frac{c_y + \frac{w}{2} - k_y}{\tilde{v}_y}; s_1 := \frac{c_x - l - k_x}{\tilde{v}_x - u_x}; s_2 := \frac{c_x - k_x}{\tilde{v}_x - u_x}; q := \frac{c_x - k_x}{\tilde{v}_x - u_x} \\
F_2 &\equiv \frac{c_x - k_x}{\tilde{v}_x - u_x} > \frac{c_x - l - k_x}{\tilde{v}_x - u_x} \\
F_3 &\equiv \left(\frac{c_y + \frac{w}{2} - k_y}{\tilde{v}_y} < \frac{c_x - l - k_x}{\tilde{v}_x - u_x} \vee \frac{c_x - k_x}{\tilde{v}_x - u_x} < \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} \vee \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} > T \vee \frac{c_x - l - k_x}{\tilde{v}_x - u_x} > T \right)
\end{aligned}$$

$$\begin{array}{c}
\frac{\frac{\frac{\exists \tilde{v}_x, \tilde{v}_y, 0 \leq r \leq T. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge F_2 \wedge F_3 \wedge p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z))}{\exists \tilde{v}_x, \tilde{v}_y, \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge F_2 \wedge F_3 \wedge \exists 0 \leq r \leq T. p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z))}}{ax} \\
\langle \rangle, sub, \mathbb{R} \quad \frac{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge F_2 \wedge F_3 \wedge \{\zeta_7\} \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})}{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge F_2 \wedge \{\zeta_6\} \langle q := s_1, s_1 := s_2; s_2 := q \rangle \langle \delta_2 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})} \\
\langle \rangle, \langle := \rangle, \langle ? \rangle, sub
\end{array}$$

Reduction of Z_4 :

Let

$$\begin{array}{c}
F_4 \equiv \left(\frac{c_y + \frac{w}{2} - k_y}{\tilde{v}_y} < \frac{c_x - k_x}{\tilde{v}_x - u_x} \vee \frac{c_y - l - k_y}{\tilde{v}_x - u_x} < \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} > T \vee \frac{c_x - k_x}{\tilde{v}_x - u_x} > T \right) \\
\frac{\frac{\frac{\exists \tilde{v}_x, \tilde{v}_y, 0 \leq r \leq T. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \neg F_2 \wedge F_4 \wedge p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z))}{\exists \tilde{v}_x, \tilde{v}_y, \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \neg F_2 \wedge F_4 \wedge \exists 0 \leq r \leq T. p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z))}}{ax} \\
\langle \rangle, sub, \mathbb{R} \quad \frac{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \neg F_2 \wedge F_4 \wedge \{\zeta_6\} \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})}{\exists \tilde{v}_x, \tilde{v}_y. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \left(\frac{c_x - k_x}{\tilde{v}_x - u_x} \leq \frac{c_x - l - k_x}{\tilde{v}_x - u_x} \right) \wedge \{\zeta_6\} \langle \delta_2 \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})} \\
\langle ? \rangle, sub
\end{array}$$

57

Proof of Y_1 :

Let

$$\begin{array}{l}
W_1 \equiv \exists 0 \leq r \leq T. p((k_x, k_y, h + 1), (c_x + u_x \cdot r, c_y, c_z)) \\
W_2 \equiv \exists \tilde{v}_x, \tilde{v}_y, 0 \leq r \leq T. (F_1 \wedge p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z))) \\
W_3 \equiv \exists \tilde{v}_x, \tilde{v}_y, 0 \leq r \leq T. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge F_2 \wedge F_3 \wedge p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z)) \\
W_4 \equiv \exists \tilde{v}_x, \tilde{v}_y, 0 \leq r \leq T. \tilde{v}_y > 0 \wedge \tilde{v}_x \neq u_x \wedge \neg F_2 \wedge F_4 \wedge p((k_x + \tilde{v}_x \cdot r, k_y + \tilde{v}_y \cdot r, 0), (c_x + u_x \cdot r, c_y, c_z))
\end{array}$$

Now, we can continue on with the first proof:

$$\begin{array}{c}
\frac{\forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \mathbf{c})) \rightarrow (\neg(A \wedge B) \vee (A \wedge B \wedge p(\mathbf{k}, \mathbf{c})))}{\rightarrow} \\
\frac{\forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \mathbf{c})) \rightarrow (A \wedge B \rightarrow p(\mathbf{k}, \mathbf{c}))}{\forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee \{t := 0\} \langle \alpha \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c}) \vee \{t := 0\} \langle \beta \rangle \langle \epsilon \rangle p(\mathbf{k}, \mathbf{c})) \rightarrow p(\mathbf{k}, \mathbf{c})) \rightarrow (A \wedge B \rightarrow p(\mathbf{k}, \mathbf{c}))}
\end{array}$$

As a reminder, we have

$$\begin{aligned}
A &\equiv k_x = 0 \wedge k_y = 0 \wedge k_z = 0 \wedge v_x = 0 \wedge v_y = 0 \wedge v_z = 0 \wedge 0 < c_y - \frac{w}{2} \wedge c_y + \frac{w}{2} < H \wedge c_z = 0 \wedge u_x > 0 \\
B &\equiv H > 0 \wedge w > 0 \wedge h \geq 0 \wedge l \geq 0 \wedge T > 0 \\
F_1 &\equiv \tilde{v}_y > 0 \wedge \tilde{v}_x = u_x \wedge \left(k_x < c_x - l \vee c_x < k_x \vee \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} > T \right) \\
F_2 &\equiv \frac{c_x - k_x}{\tilde{v}_x - u_x} > \frac{c_x - l - k_x}{\tilde{v}_x - u_x} \\
F_3 &\equiv \left(\frac{c_y + \frac{w}{2} - k_y}{\tilde{v}_y} < \frac{c_x - l - k_x}{\tilde{v}_x - u_x} \vee \frac{c_x - k_x}{\tilde{v}_x - u_x} < \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} \vee \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} > T \vee \frac{c_x - l - k_x}{\tilde{v}_x - u_x} > T \right) \\
F_4 &\equiv \left(\frac{c_y + \frac{w}{2} - k_y}{\tilde{v}_y} < \frac{c_x - k_x}{\tilde{v}_x - u_x} \vee \frac{c_x - l - k_x}{\tilde{v}_x - u_x} < \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} \vee \frac{c_y - \frac{w}{2} - k_y}{\tilde{v}_y} > T \vee \frac{c_x - k_x}{\tilde{v}_x - u_x} > T \right)
\end{aligned}$$

If the left hand side $\forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \mathbf{c}))$ implies $\neg(A \wedge B)$, then there is nothing to show, so we may instead simply prove

$$A \wedge B \wedge \forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \mathbf{c})) \rightarrow p(\mathbf{k}, \mathbf{c})$$

Then the proof continues as:

$$\begin{array}{c}
A \wedge B \wedge \forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \mathbf{c})) \rightarrow p((0, 0, 0), (c_x, c_y, 0)) \\
\hline
eq \quad A \wedge B \wedge \forall \mathbf{k}, \mathbf{c} ((k_y \geq H \vee W_1 \vee W_2 \vee W_3 \vee W_4) \rightarrow p(\mathbf{k}, \mathbf{c})) \rightarrow p(\mathbf{k}, \mathbf{c})
\end{array}$$

Now, we can consider the W_i 's as "rules" that decide which values of \mathbf{k} and \mathbf{c} makes $p(\mathbf{k}, \mathbf{c})$ true, and prove efficiency by working backwards. Initially, the chicken starts at $(0, 0, 0)$ and the car at $(c_x, c_y, 0)$. It is not too hard to see that the chicken can choose to hover in place until the car passes it (using rule W_1 and the fact that $u_x > 0$), at which point the chicken can exercise rule W_3 (it is easy to check that in the case where the x -component of the position of the car is greater than that of the chicken, F_3 is true; F_2 is always true since the chicken cannot move in the negative y -direction; finally, the chicken can simply choose its x -velocity v_x to be 0, which will satisfy $u_x \neq v_x$) until it reaches $y = H$. This shows that $p(0, 0, 0, c_x, c_y, 0)$ is true.

This proves that in this simple version of the "chicken-crossing-the-road" game, the chicken will always have a way to cross the road.

Note that a very easy modification of the program and proof above can allow us to place more restrictions on what the chicken can do and/or generalize on what the cars can do: for example, it is very easy to place a maximum and minimum on the components of the chicken's velocity with little extra work or to allow the cars to travel with any velocity (not just positive)